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THE POTENTIAL-ENERGY BUDGET OF THE STRATOSPHERE OVER NORTH AMERICA DURING THE WARMING OF 1957

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SCIENTIFIC REPORT No. 3
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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
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ABSTRACT

The evaluation of the terms in the balance equation for potential and internal energy, was carried out using the 100-mb, 50-mb and 25-mb values of geopotential, temperature, the geostrophic wind components and adiabatic vertical motion at each point of a grid covering most of the North American continent and the adjacent North Atlantic. The investigation considered a limited volume in which changes in potential and internal energy (alternatively referred to as total potential energy) were affected by flux of energy across the boundaries.

Terms in the total potential-energy budget explicitly involving lifting or lowering of the mean height of the boundary pressure surfaces were found to be relatively unimportant. Conversion of energy within the volume was from kinetic to potential energy throughout most of the period.

The volume gained total potential energy through vertical transport of enthalpy across the upper boundary surface and lost total potential energy through transport across the lower boundary surface. The net effect was a gain in total potential energy during the early part of the warming period and a loss during the latter part. The contributions to total potential energy changes in layers of limited depth due to net vertical flux of enthalpy can be quite significant.

During the early stages of the warming, the volume lost total potential energy through net horizontal flux across the vertical boundaries. But during the period of pronounced warming throughout the volume, the same process brought about a large gain in total potential energy.

The values of the horizontal and vertical fluxes of energy, the conversion rates and the changes due to mean lifting or lowering of the

mass considered, were balanced against the local change of total potential energy. The residual was taken as an approximate estimate of the rate of addition of heat due to diabatic processes. However, these estimates are not considered very reliable since they include any systematic errors in the computations.

1. INTRODUCTION

In a previous scientific report (Lateef, 1963), the author has discussed in detail, the kinetic-energy budget of the stratosphere over North America during the warming of mid-January to mid-February, 1957. The present report continues the discussion of the energy budget for the same atmospheric region and time period and is intended to be read in conjunction with the previous report. Many of the details of the analysis of available data and computational procedures, as well as descriptions of the evolution of the height, temperature and vertical motion fields during the course of the warming, are already contained in previous published reports (Craig, Lateef and Mitchem, 1961a, 1961b; Craig and Lateef, 1961, 1962; Lateef, 1963). Hence, it is not considered essential to repeat all of the detail presented therein and the present report is confined to a discussion of the terms involved in the potential plus internal-energy balance, with specific references wherever necessary to the previous reports.

The derivation of the balance equation for potential plus internal energy for a limited atmospheric volume and the finite-difference approximations used to estimate the terms in the equation are presented in sections 2 and 3.

The rest of the report is devoted to a discussion of the computed values of the terms and the relative importance of the physical processes represented by these terms in contributing to changes in potential plus internal energy of the volume under study.

2. THE POTENTIAL PLUS INTERNAL ENERGY BALANCE EQUATION

The potential energy dP, per unit horizontal area of an atmospheric layer of mass dp/g is gZ (dp/g) or Z dp, where Z is the mean elevation of the mass above sea level, g is acceleration of gravity and dp is the difference in pressure between the top and bottom surfaces of the layer. The potential energy P, for the volume under study, or strictly speaking for the mass of the atmosphere between the two constant-pressure surfaces p_0 (= 100 mb) and p (= 25 mb) and the vertical boundaries, is given by

$$P = \int_p^{p_0} \int_A dA \, dP = \int_p^{p_0} \int_A dA \, Z \, dp \tag{1}$$

The right side of (1) can be re-arranged as

$$\int_p^{p_0} \int_A dA \, d\,(pZ) - \int_p^{p_0} \int_A dA \, p \, dZ$$

Using the hydrostatic relation $dp = - g\rho \, dZ$ and the equation of state $p = \rho RT$, we may write

$$P = p_0 \left(\int_A Z \, dA \right)_{p_0} - p \left(\int_A Z \, dA \right)_p + \int_p^{p_0} \int_A RT \, dA \, \frac{dp}{g} \tag{2}$$

The internal energy, I, for the same atmospheric mass is given by

$$I = \int_p^{p_0} \int_A c_v \, T \, dA \, \frac{dp}{g} \tag{3}$$

Combining (2) and (3), and using the relationship $c_v + R = c_p$, we write

$$P + I = p_0 \left(\int_A Z \, dA\right)_{p_0} - p \left(\int_A Z \, dA\right)_p + \int_p^{p_0} \int_A c_p \, T \, dA \, \frac{dp}{g} \qquad (4)$$

The first two terms on the right side of (4) represent the portion of potential energy which depends on the difference in geopotential height of the pressure surfaces. The third term which is a function of the temperature distribution within the chosen constant atmospheric mass, involves the remaining part of the potential energy as well as the internal energy. It is easily seen that the potential energy included in this term bears the constant ratio R/c_v to the internal energy. If the integration were carried out over the entire depth of the atmosphere, the first two terms would vanish and the potential energy of the layer would always be less than the internal energy by the factor R/c_v. For a limited atmospheric mass, however, such a simple relationship between the potential and internal energies would not be valid. In what follows, we shall speak of P + I, as the <u>total</u> potential energy of the volume.

Derivation

The mathematical expression that represents the balance equation for the total potential energy can be derived by differentiation of (4) with respect to time. Thus,

$$\frac{\partial}{\partial t}(P + I) = p_0 \frac{\partial}{\partial t} \left(\int_A Z \, dA\right)_{p_0} - p \frac{\partial}{\partial t} \left(\int_A Z \, dA\right)_p + c_p \int_p^{p_0} \int_A \frac{\partial T}{\partial t} \, dA \, \frac{dp}{g} \qquad (5)$$

If we use the first law of thermodynamics in the form $dq/dt = c_p \, dT/dt - \alpha\omega$, where dq represents an increment in specific heat added in time dt, we may express $\partial T/\partial t$ in the form,

$$c_p \frac{\partial T}{\partial t} = - c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \frac{\partial T}{\partial p} \right) + \omega \alpha + \frac{dq}{dt} \tag{6}$$

Substitution of (6) into the last term of (5) and the use of the two-dimensional form of the divergence theorem, yields

$$c_p \int_p^{p_0} \int_A \frac{\partial T}{\partial t} \, dA \, \frac{dp}{g} = - c_p \int_p^{p_0} \left(\oint_L T v_n \, dL \right) \frac{dp}{g} - c_p \int_p^{p_0} \int_A \omega \frac{\partial T}{\partial p} \, dA \, \frac{dp}{g} +$$

$$c_p \int_p^{p_0} \int_A T \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dA \, \frac{dp}{g} + \int_p^{p_0} \int_A \omega \alpha \, dA \, \frac{dp}{g} + \int_p^{p_0} \int_A \frac{dq}{dt} \, dA \, \frac{dp}{g} \tag{7}$$

where v_n is the horizontal wind component perpendicular to the vertical boundary of the volume, positive outward and L is the perimeter of the analysis area A.

Using the continuity equation, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \frac{\partial \omega}{\partial p}$, and re-arranging the term $\omega \frac{\partial T}{\partial p}$ as $\frac{\partial}{\partial p} (T\omega) - T \frac{\partial \omega}{\partial p}$, we may finally write

$$\frac{\partial}{\partial t} (P + I) = \frac{\partial}{\partial t} \left[p_0 \left(\int_A Z \, dA \right)_{p_0} - p \left(\int_A Z \, dA \right)_p \right] - \frac{1}{g} \int_p^{p_0} \left(\oint_L c_p T v_n \, dL \right) dp$$

$$+ \frac{1}{g} \left[\left(\int_A c_p T\omega \, dA \right)_p - \left(\int_A c_p T\omega \, dA \right)_{p_0} \right] + \frac{R}{g} \int_p^{p_0} \left(\int_A \frac{\omega T}{p} \, dA \right) dp$$

$$+ \int_p^{p_0} \int_A \frac{dq}{dt} \, dA \, \frac{dp}{g} \tag{8}$$

Physical meaning of the terms

The term $\frac{\partial}{\partial t} (P + I)$ on the left side of (8) is the time-change of the total potential energy in the volume bounded by the perimeter L and

the two specific pressure levels p_0 and p.

The physical processes which bring about changes in P + I, are represented by the terms on the right side of (8). Thus, the total potential energy may be changing as a result of:

(a) Net vertical displacements of the boundary pressure surfaces. This is represented by the term

$$\left(p \frac{\partial z}{\partial t}\right) = \left[\frac{\partial}{\partial t} p_0 \left(\int_A z \; dA\right)_{p_0}\right] - \left[\frac{\partial}{\partial t} p \left(\int_A z \; dA\right)_p\right]$$

A positive value for either term within brackets on the right side, indicates a mean lifting of the corresponding pressure surface and thus an increase in the mean elevation of the mass being considered. The net contribution to the potential energy change is, however, given by the relative effects of the lifting or lowering of the boundary pressure surfaces.

(b) Advection of potential plus internal energy or alternatively, advection of enthalpy into the volume across the vertical boundary. This is represented by the term

$$\left(c_p \; Tv_n\right) = - \frac{1}{g} \int_p^{p_0} \left(\oint_L c_p \; Tv_n \; dL\right) dp$$

(c) Advection of enthalpy into the volume through the boundary pressure surfaces. This is represented by the term

$$\left(c_p \; T\omega\right) = \frac{1}{g}\left[\left(\int_A c_p \; T\omega \; dA\right)_p - \left(\int_A c_p \; T\omega \; dA\right)_{p_0}\right]$$

(d) Generation of total potential energy within the volume by conversion from kinetic energy. This is represented by the term

$$(\omega\alpha) = \frac{R}{g} \int_{p}^{p_0} \left(\int_A \frac{\omega T}{p} \, dA \right) dp$$

(e) Rate of addition of heat to the atmospheric mass being considered, through diabatic processes. This is represented by the term

$$(Q) = \int_{p}^{p_0} \int_A \frac{dq}{dt} \, dA \, \frac{dp}{g}$$

Since this term is actually evaluated as residual out of the imbalance among the other terms, it also includes any errors in the computations.

3. COMPUTATIONAL PROCEDURES

The details of procedures to obtain derived quantities such as vertical motion and of approximations to area and line integrals have already been presented in the previous report (Lateef, 1963). Only the finite-difference approximations used for evaluating the terms in the total potential energy balance equation will, therefore, be given here.

$$(P + I) \doteq (100 \text{ mb}) \left(\sum_{i=1}^{90} Z_i \, \Delta A_i \right)_{100 \text{ mb}} - (25 \text{ mb}) \left(\sum_{i=1}^{90} Z_i \, \Delta A_i \right)_{25 \text{ mb}}$$

$$+ (12.5 \text{ mb}) \frac{c_p}{g} \left[\left(\sum_{i=1}^{90} T_i \, \Delta A_i \right)_{25 \text{ mb}} + 3 \left(\sum_{i=1}^{90} T_i \, \Delta A_i \right)_{50 \text{ mb}} + 2 \left(\sum_{i=1}^{90} T_i \, \Delta A_i \right)_{100 \text{ mb}} \right] \tag{9}$$

$$\frac{1}{g} \int_{p}^{p_0} \left(\oint_L c_p \, Tv_n \, dL \right) dp \doteq (12.5 \text{ mb}) \frac{c_p}{g} \left[\left(\sum_{j=1}^{38} \left(Tv_n \right)_j \Delta L_j \right)_{25 \text{ mb}} + \right.$$

$$\left. 3 \left(\sum_{j=1}^{38} \left(Tv_n \right)_j \Delta L_j \right)_{50 \text{ mb}} + 2 \left(\sum_{j=1}^{38} \left(Tv_n \right)_j \Delta L_j \right)_{100 \text{ mb}} \right] \tag{10}$$

$$\frac{1}{g}\left(\left(\int_A c_p\ T\omega\ dA\right)_p - \left(\int_A c_p\ T\omega\ dA\right)_{p_0}\right) \doteq$$

$$\frac{c_p}{g}\left[\left(\sum_{i=1}^{90} (T\omega)_i\ \Delta A_i\right)_{25\ mb} - \left(\sum_{i=1}^{90} (T\omega)_i\ \Delta A_i\right)_{100\ mb}\right] \qquad (11)$$

The $(\omega\alpha)$ term in the equation is the same as the (ωT) term defined in the previous report (Lateef, 1963) as the energy conversion term but with opposite sign. Time derivatives were approximated by 24-hour changes centered at map time.

All computations were carried out on the IBM 709 computer at Florida State University.

Discussions concerning the specific results obtained in the evaluation of the various terms for twice daily maps, at 0300 GCT and 1500 GCT for the period 0300 GCT of 17 January 1957 to 1500 GCT of 14 February 1957, are contained in the following sections.

4. VARIATION OF TOTAL POTENTIAL ENERGY

The volume integral of the total potential energy given by equation (9) was evaluated twice daily for the entire volume as well as for the 100-50 mb and 50-25 mb layers. Figure 1 shows the variation of P + I with time during the period while Figures 2 and 3 show the variation of the total potential energy within the layers. The values of P + I have been somewhat smoothed by the application of a binomial smoothing function with weights 1/4, 1/2, 1/4.

The time variations of P + I, exhibited in these figures are almost similar in nature. Increases and decreases in total potential energy both

for the volume and for the two layers treated separately, occurred more or less simultaneously. The variations of (P + I) during the four time periods (specified in the previous report) may be described as follows.

(a) From 17 January to about 25 January, P + I showed a gradual decrease. During the same period the total kinetic energy gradually increased in spite of shorter period fluctuations. Pronounced warming had not begun to affect the volume.

(b) From 25 January to about 4 February, P + I showed a gradual increase with shorter period fluctuations superimposed, corresponding to a general decrease in the total kinetic energy. Warming was evident at 25 mb, but not yet throughout the volume.

(c) From 4 February to about 9 February, P + I increased very rapidly while there was a simultaneous rapid decrease in the total kinetic energy. This was the period when rapid temperature changes were taking place at 100 mb and 50 mb.

(d) From 9 February until near the end of the period, P + I began to decrease. At the same time, the total kinetic energy was also decreasing though at a lesser rate than previously. This was the period when the rapid temperature changes had already occurred and the north-south temperature gradient at all levels was reversed from that prevailing in mid-January.

The values of P + I, as well as those of P and I separately, are listed in Table I.

Table 1. P, I and P + I for the volume, in units of 10^{26} ergs.

Date	Time (GCT)	P	I	P + I
17 January	0300	396.02	322.07	718.09
	1500	396.02	321.22	717.24
18	0300	394.52	321.99	716.51
	1500	394.65	320.63	715.28
19	0300	393.41	320.54	713.95
	1500	393.51	319.18	712.69
20	0300	392.73	318.74	711.47
	1500	392.77	317.55	710.32
21	0300	391.91	317.20	709.11
	1500	391.84	316.25	708.09
22	0300	391.81	315.74	707.55
	1500	391.68	315.59	707.27
23	0300	391.63	315.69	707.31
	1500	392.03	315.75	707.78
24	0300	391.67	316.74	708.41
	1500	392.02	316.99	709.02
25	0300	391.61	317.71	709.32
	1500	391.40	317.91	709.31
26	0300	390.99	318.55	709.54
	1500	392.18	317.82	710.00
27	0300	392.17	318.01	710.18
	1500	392.17	317.77	709.94
28	0300	391.74	318.27	710.01
	1500	393.09	317.53	710.62
29	0300	392.97	318.22	711.19
	1500	393.32	317.96	711.28
30	0300	392.43	318.79	711.22
	1500	392.81	318.92	711.73
31	0300	392.97	319.79	712.77
	1500	393.79	320.13	713.92
1 February	0300	393.85	321.35	715.20
	1500	395.20	321.06	716.26
2	0300	394.50	322.41	716.91
	1500	395.09	322.68	717.77
3	0300	395.02	323.54	718.56
	1500	395.39	323.70	719.09
4	0300	394.82	325.47	720.29
	1500	396.78	325.40	722.18
5	0300	396.51	327.34	723.85
	1500	397.40	328.17	725.57
6	0300	397.50	330.18	727.68
	1500	398.89	330.62	729.51
7	0300	398.17	332.44	730.61
	1500	398.67	332.88	731.55
8	0300	398.93	333.75	732.67
	1500	400.27	333.10	733.37

Table 1. Continued

Date	Time (GCT)	P	I	P + I
9 February	0300	399.08	334.23	733.31
	1500	399.67	333.51	733.18
10	0300	399.53	333.47	733.00
	1500	399.74	332.87	732.61
11	0300	400.09	331.92	732.01
	1500	399.56	331.61	731.17
12	0300	399.43	331.11	730.54
	1500	399.76	330.16	729.92
13	0300	399.27	330.41	729.68
	1500	400.51	329.46	729.97
14	0300	400.37	329.74	730.11
	1500	401.13	329.27	730.40

From these values, the magnitudes of $\frac{\partial P}{\partial t}$, $\frac{\partial I}{\partial t}$ and $\frac{\partial (P + I)}{\partial t}$ computed for the four periods mentioned before, are given in Table 2.

Table 2. Average values of $\frac{\partial P}{\partial t}$, $\frac{\partial I}{\partial t}$ and $\frac{\partial (P + I)}{\partial t}$, in units of 10^{24} ergs per hour.

Period	$\frac{\partial P}{\partial t}$	$\frac{\partial I}{\partial t}$	$\frac{\partial (P + I)}{\partial t}$
17 January to 25 January	-2.30	-2.26	-4.57
25 January to 4 February	1.34	3.22	4.56
4 February to 9 February	3.55	6.98	10.53
9 February to 14 February	1.56	-3.99	-2.44

The values of P, I and P + I, listed in Table 1, are two orders of magnitude higher than the corresponding values of K (Lateef, 1963). The rates of change of P, I and P + I (Table 2) are an order of magnitude larger than the rate of change of K in the volume. This is in agreement with the relative magnitudes of $\frac{\partial (P + I)}{\partial t}$ and $\frac{\partial K}{\partial t}$ computed by Barnes (1962) for the

100-30 mb layer over the Northern Hemisphere. It must be pointed out that the values of $\frac{\partial}{\partial t}(P + I)$ do not represent rates of change of potential energy 'available' for conversion to kinetic energy under adiabatic motion. No attempt has been made in the present investigation to estimate the ratio of available to total potential energy in the volume. Computations carried out by Reed (1962) at the 50-mb level over a large portion of the Northern Hemisphere for the period 25 January to 9 February, 1957, indicate that the rates of change of available potential energy on a hemispheric scale were at least an order of magnitude smaller than the rates of change of potential energy in the limited volume under study.

5. POTENTIAL-ENERGY CHANGES DUE TO NET MOVEMENT OF THE BOUNDARY PRESSURE SURFACES

Change in potential energy brought about by the net lifting or lowering of the mean heights above sea level of the top and bottom surfaces of the atmospheric mass being considered, is represented by the $\left(p\,\frac{\partial z}{\partial t}\right)$ term of Section 2. Computations of such changes due to mean lifting or lowering of the 25-mb and 100-mb surfaces were made according to the finite difference procedure mentioned in Section 3. The values of the $\left(p\,\frac{\partial z}{\partial t}\right)$ term at each map time are listed in Table 3.

Table 3. Rate of change of potential energy due to the $\left(p\,\frac{\partial z}{\partial t}\right)$ process, in units of 10^{21} ergs per second.

Date	Time (GCT)	$\left(p\,\frac{\partial z}{\partial t}\right)$
17 January	0300	-0.83
	1500	-1.19
18	0300	-1.07
	1500	-0.73

Table 3. Continued

Date	Time (GCT)	$\left(p \frac{\partial z}{\partial t}\right)$
19 January	0300	-0.43
	1500	-0.16
20	0300	-0.13
	1500	-0.25
21	0300	-0.09
	1500	0.21
22	0300	0.19
	1500	0.04
23	0300	-0.02
	1500	-0.33
24	0300	-0.59
	1500	-0.73
25	0300	-0.92
	1500	-0.94
26	0300	0.54
	1500	0.97
27	0300	0.33
	1500	0.10
28	0300	0.75
	1500	0.94
29	0300	0.16
	1500	-0.53
30	0300	-0.62
	1500	-0.08
31	0300	0.35
	1500	0.55
1 February	0300	0.68
	1500	0.25
2	0300	-0.31
	1500	-0.26
3	0300	-0.26
	1500	-0.29
4	0300	0.29
	1500	0.53
5	0300	0.03
	1500	0.11
6	0300	0.07
	1500	-0.28
7	0300	-0.55
	1500	0.20
8	0300	0.82
	1500	0.23
9	0300	0.15
	1500	0.30
10	0300	0.73
	1500	0.87
11	0300	0.48
	1500	0.15

Table 3. Continued

Date	Time (GCT)	$\left(p \frac{\partial z}{\partial t}\right)$
12 February	0300	0.41
	1500	0.65
13	0300	0.97
	1500	1.26
14	0300	1.21
	1500	1.19

The net contributions to rate of change of potential energy due to the $\left(p \frac{\partial z}{\partial t}\right)$ process, during the four periods, are given in Table 4.

Table 4. Average values of $\left(p \frac{\partial z}{\partial t}\right)$, in units of 10^{24} ergs per hour.

Period	$\left(p \frac{\partial z}{\partial t}\right)$
17 January to 25 January	-1.36
25 January to 4 February	0.16
4 February to 9 February	0.57
9 February to 14 February	2.80

According to the computations carried out by Barnes (1962), the $\left(p \frac{\partial z}{\partial t}\right)$ process contributed significantly to rate of change of potential energy in the 100-30 mb layer over the Northern Hemisphere. For the limited volume used in this report, however, the $\left(p \frac{\partial z}{\partial t}\right)$ term was relatively unimportant compared to the advection and conversion terms of equation (8).

6. TOTAL POTENTIAL-ENERGY CHANGES DUE TO HORIZONTAL ADVECTION OF ENTHALPY

The contribution to the rate of change of potential plus internal energy in the volume due to advection of such energy across the vertical boundaries is represented by the $(c_p\ Tv_n)$ term specified in Section 2. The rate of change of total potential energy in the volume due to the $(c_p\ Tv_n)$ process was computed according to the finite difference summation procedure shown in equation (10). The values of T at the grid points just inside the analysis area were assigned to the perimeter increments. The error in the evaluation of the term $\Sigma\ (c_p\ Tv_n)_j\ \Delta L_j$, in equation (10), at any pressure level, by not using the T's along the perimeter itself, amounted to less than two per cent, even under assumption of extreme horizontal variations in temperature.

As in the case of the $(\phi\ v_n)$ term described in the previous report (Lateef, 1963), the flux of enthalpy due to both the geostrophic and ageostrophic wind components was computed in order to obtain a more realistic estimate of the $(c_p\ Tv_n)$ process. The procedure adopted for the computations, using the values of horizontal divergence at the grid points, was essentially similar to that explained in detail in the previous report referred to. The temperature values at 75-mb and 37.5-mb levels were taken as the mean of T at 100 mb and 50 mb and the mean of T at 50 mb and 25 mb respectively.

The error introduced by neglecting the $(c_p\ \widetilde{T'v'}_{na})$ term in the $(c_p\ Tv_n)$ process, was estimated to be negligible compared to the size of the $(c_p\ Tv_{ng})$ term. The contribution of the $(c_p\ \tilde{T}\ \tilde{v}_{na})$ term to the horizontal flux of enthalpy was found to be significant and indicated that estimates based on the geostrophic flux alone would, in most cases, be in error.

Table 5 gives the values of the (c_p Tv_{ng}) term, the (c_p Tv_{na}) term and the sum of these two terms representing the rate of change of total potential-energy in the volume due to the (c_p Tv_n) process.

Table 5. Rate of change of total potential-energy of the volume due to the (c_p Tv_n) process, in units of 10^{21} ergs per second.

Date	Time (GCT)	(c_p Tv_{ng})	(c_p Tv_{na})	(c_p Tv_n)
17 January	0300	-19.25	14.21	-5.04
	1500	-9.60	1.24	-8.36
18	0300	-10.35	2.43	-7.92
	1500	-1.28	-5.11	-6.39
19	0300	-2.73	1.60	-1.13
	1500	9.29	-11.43	-2.14
20	0300	0.45	-6.79	-6.34
	1500	7.16	-9.17	-2.01
21	0300	5.10	-8.06	-2.96
	1500	11.68	-20.96	-9.28
22	0300	10.98	-20.93	-9.95
	1500	9.09	-20.33	-11.24
23	0300	-1.36	-13.13	-14.49
	1500	-1.01	-14.30	-15.31
24	0300	0.99	-12.39	-11.40
	1500	3.70	-10.22	-6.52
25	0300	-3.17	-2.82	-5.99
	1500	-8.67	-3.34	-12.01
26	0300	-10.34	-1.81	-12.15
	1500	-11.28	-9.88	-21.16
27	0300	-17.74	-3.27	-21.01
	1500	-31.51	7.74	-23.77
28	0300	-28.75	0.44	-28.31
	1500	-21.59	-9.27	-30.86
29	0300	-22.83	-5.40	-28.23
	1500	-16.45	-9.59	-26.04
30	0300	-21.95	-0.73	-22.68
	1500	-13.30	-10.33	-23.63
31	0300	-28.59	2.67	-25.92
	1500	-23.63	-2.92	-26.55
1 February	0300	-21.30	-5.33	-26.63
	1500	-17.92	-10.30	-28.22
2	0300	-18.94	-3.03	-21.97
	1500	-10.67	-4.68	-15.35
3	0300	-3.95	-11.64	-15.59
	1500	9.69	-23.16	-13.47

Table 5. Continued

Date	Time (GCT)	$(c_p\ Tv_{ng})$	$(c_p\ Tv_{na})$	$(c_p\ Tv_n)$
4 February	0300	17.88	-20.76	-2.88
	1500	20.61	-15.77	4.84
5	0300	21.17	-14.52	6.65
	1500	26.61	-18.20	8.41
6	0300	27.13	-11.60	15.53
	1500	27.24	-10.39	16.85
7	0300	24.96	-11.15	13.81
	1500	30.76	-15.50	15.26
8	0300	19.95	1.85	21.80
	1500	18.83	2.89	21.72
9	0300	11.18	7.98	19.16
	1500	9.19	3.28	12.47
10	0300	12.46	-0.02	12.44
	1500	19.29	-0.69	18.60
11	0300	14.33	11.70	26.03
	1500	20.24	4.42	24.66
12	0300	9.22	13.42	22.64
	1500	12.01	9.43	21.44
13	0300	8.81	20.35	29.16
	1500	10.92	19.68	30.60
14	0300	10.92	18.15	29.07
	1500	14.35	14.21	28.56

The rates of change of total potential-energy in the volume during the four periods due to the $(c_p\ Tv_n)$ process are shown in Table 6.

Table 6. Average values of $(c_p\ Tv_n)$, in units of 10^{24} ergs per hour.

Period	$(c_p\ Tv_n)$
17 January to 25 January	-27.33
25 January to 4 February	-77.04
4 February to 9 February	47.84
9 February to 14 February	82.11

7. TOTAL POTENTIAL-ENERGY CHANGES DUE TO VERTICAL ADVECTION OF ENTHALPY

The rates of change of total potential energy in the volume as well as in the 100-50 mb and 50-25 mb layers due to the (c_p Tω) process were computed according to the finite difference summation procedure shown in equation (11). The results of the computations are shown in Table 7.

Table 7. Rate of change of total potential-energy of the volume due to the (c_p Tω) process, in units of 10^{21} ergs per second.

Date	Time (GCT)	(c_p Tω) 100-50 mb	(c_p Tω) 50-25 mb	(c_p Tω) Volume
17 January	0300	0.90	4.68	5.58
	1500	3.23	5.13	8.36
18	0300	2.92	3.83	6.75
	1500	-1.39	3.66	2.27
19	0300	-6.35	4.06	-2.29
	1500	-4.90	4.27	-0.63
20	0300	-1.69	2.67	0.97
	1500	0.99	1.69	2.67
21	0300	0.17	2.55	2.72
	1500	1.30	1.49	2.79
22	0300	4.08	-0.37	3.71
	1500	5.92	-0.27	5.65
23	0300	9.48	-0.33	9.15
	1500	9.37	-0.09	9.28
24	0300	5.88	-0.06	5.82
	1500	-1.45	-0.04	-1.49
25	0300	-2.83	-0.49	-3.31
	1500	1.50	-1.77	-0.26
26	0300	6.71	-0.99	5.72
	1500	9.84	-1.02	8.82
27	0300	8.52	-0.05	8.47
	1500	7.90	2.58	10.49
28	0300	13.31	2.76	16.07
	1500	16.23	2.16	18.40
29	0300	11.48	4.39	15.86
	1500	6.84	5.98	12.83
30	0300	6.97	7.01	13.99
	1500	6.31	7.38	13.69
31	0300	8.31	6.38	14.70
	1500	9.51	4.79	14.30

Table 7. Continued

Date	Time (GCT)	$(c_p\ T\omega)$ 100-50 mb	$(c_p\ T\omega)$ 50-25 mb	$(c_p\ T\omega)$ Volume
1 February	0300	10.40	2.76	13.16
	1500	8.18	2.01	10.20
2	0300	6.26	1.71	7.97
	1500	2.71	-2.85	-0.14
3	0300	4.41	-4.97	-0.57
	1500	-0.03	-3.86	-3.88
4	0300	-9.91	-5.00	-14.91
	1500	-14.91	-8.31	-23.21
5	0300	-13.91	-11.57	-25.49
	1500	-13.30	-17.37	-30.67
6	0300	-17.99	-15.16	-33.15
	1500	-17.75	-16.52	-34.26
7	0300	-14.65	-20.52	-35.17
	1500	-12.00	-24.19	-36.19
8	0300	-13.54	-25.49	-39.04
	1500	-13.50	-25.96	-39.47
9	0300	-9.92	-24.51	-34.43
	1500	-6.72	-19.59	-26.32
10	0300	-8.74	-15.90	-24.65
	1500	-12.06	-16.03	-28.09
11	0300	-18.45	-16.91	-35.37
	1500	-23.74	-12.77	-36.50
12	0300	-24.53	-13.38	-37.91
	1500	-25.42	-13.43	-38.85
13	0300	-24.35	-13.00	-37.35
	1500	-27.33	-10.83	-38.16
14	0300	-24.25	-10.37	-34.62
	1500	-22.08	-12.79	-34.87

As in the case of the $(\phi\omega)$ process discussed in the previous report (Lateef, 1963), the values of the $(c_p\ T\omega)$ term, listed in Table 7 show that, in general, the volume gained potential and internal energy from the beginning of the period until about 3 February. It consistently lost potential and internal energy from 3 February onwards. During both these periods, the volume gained energy owing to transport through the 25-mb surface and lost energy owing to transport through the 100-mb surface. However, in the earlier part of the warming period, the gain

exceeded the loss; in the later part, the loss exceeded the gain. This is illustrated in Fig. 4, in which the values of $\Sigma\,(c_p\ T\omega)_i\ \Delta A_i$, at each level, are plotted for each map time.

The rates of change of total potential-energy in the volume during the four subperiods due to the $(c_p\ T\omega)$ process, are shown in Table 8.

Table 8. Average values of $(c_p\ T\omega)$, in units of 10^{24} ergs per hour.

Period	$(c_p\ T\omega)$
17 January to 25 January	12.79
25 January to 4 February	30.72
4 February to 9 February	-115.68
9 February to 14 February	-121.89

8. TOTAL POTENTIAL-ENERGY CHANGES DUE TO CONVERSION FROM KINETIC ENERGY

The rates of change of total potential energy in the volume due to conversion from kinetic energy brought about by adiabatic vertical motions have already been discussed in the previous report (Lateef, 1963). For the sake of completeness and immediate reference, the values for the $(\omega\alpha)$ term are reproduced in Table 9. Figure 5 illustrates the nature and magnitude of the conversion process within the 100-50 mb and 50-25 mb layers.

Table 9. Average values of $(\omega\alpha)$, in units of 10^{24} ergs per hour.

Period	$(\omega\alpha)$
17 January to 25 January	16.55
25 January to 4 February	46.53
4 February to 9 February	74.57
9 February to 14 February	37.66

Adiabatic vertical motions contributed to an increase in the total potential energy of the volume, throughout the warming period.

9. TOTAL POTENTIAL-ENERGY BUDGET OF THE VOLUME

The average values of the terms in equation (8) for the four periods are listed in Table 10.

As mentioned previously, the term involving net lifting or lowering of the pressure surfaces proved to be negligible in the total potential-energy budget of the volume. The relatively high values of the $(c_p\ Tv_n)$ term, however, indicate that the magnitudes of the $(p \frac{\partial z}{\partial t})$, $(c_p\ T\omega)$ and $(\omega\alpha)$ terms may not be representative of the average stratosphere at all longitudes during this period.

The imbalance among the computed terms, presumably represents the rate of generation of heat by diabatic processes mainly involving absorption of radiation. The (Q) term also includes errors in the computations, particularly any systematic errors involved in areal averages of quantities. There is no obvious way of estimating these systematic errors and the values for the (Q) term should not, therefore, be regarded as

representative of actual diabatic processes. It is interesting to note that the value of (Q) when summed over the entire period, amounted to a net cooling rate of 920 ergs cm^{-2} sec^{-1} for the 100-25 mb layer or about 0.1°C per day. The magnitude of the cooling rate appears reasonable when compared with estimates of radiational heating rates for stratospheric levels over the Northern Hemisphere (Barnes, 1962).

Table 10. Average values of terms in the total potential-energy balance equation, in units of 10^{24} ergs per hour. Positive value indicates process acted to increase the total potential-energy of the volume

	17 January-25 January	25 January-4 February	4 February-9 February	9 February-14 February
$\partial (P + I)/\partial t$	-4.57	4.56	10.53	-2.44
$(p \frac{\partial z}{\partial t})$	-1.36	0.16	0.57	2.80
$(c_p Tv_n)$	-27.33	-77.04	47.84	82.11
$(c_p T\omega)_{25\ mb}$	40.69	116.74	123.66	40.90
$(c_p T\omega)_{100\ mb}$	-27.90	-86.02	-239.34	-162.79
$(c_p T\omega)$	12.79	30.72	-115.68	-121.89
(ωz)	16.55	46.53	74.57	37.66
(Q)	-5.22	4.19	3.23	-3.12

10. SUMMARY AND CONCLUSIONS

For the lower stratosphere over North America bounded by the 100- and 25-mb pressure surfaces, the terms in the potential plus internal-energy balance equation have been evaluated. Since the volume under study was an open system, energy exchanges with the surrounding atmosphere were taken into consideration. These exchanges took place through the boundary pressure surfaces as well as through the vertical boundaries.

The actual variation of total potential energy in the volume resulted from a comparatively small difference among other large terms. But while the rates of change of total potential energy were an order of magnitude higher than corresponding values for the total kinetic energy in the volume (Lateef, 1963), the contributions due to the major processes involved, were about the same order of magnitude.

As in the case of the kinetic-energy budget, the present investigation has further established the relative importance of boundary processes, namely the $(c_p\ Tv_n)$ and $(c_p\ T\omega)$ terms, in the total potential-energy budget of a limited portion of the atmosphere. The values of $\frac{\partial}{\partial t}(P + I)$ and $(\omega\alpha)$ in Table 10, also indicate that any estimates of the former based purely on conversion rates could be in error, both in magnitude and sign.

The $(p\ \frac{\partial Z}{\partial t})$ term has been found to be relatively unimportant for the limited volume under study. But the large magnitudes of the other processes imply that the values of this term may not be representative of the stratosphere at all longitudes. According to Barnes (1962) this term contributed significantly to potential-energy changes in the 100-30 mb layer over the Northern Hemisphere.

The estimate of the diabatic term (Q) has been found to be reasonable when averaged over the entire warming period. But since this term includes any systematic errors in the computations, the values for (Q) are not considered as satisfactory as those of the other terms.

It must again be pointed out that this study has been based on data over a limited region and for a short period. Evaluation of the results in terms of application on a hemispheric scale and for a long enough period is, therefore, difficult. Nevertheless, it is felt that the computational procedures used in the present investigation are reliable and fruitful. With the increase in observations in recent years, there is increasing possibility of extending the computations of energy processes over much larger regions in the stratosphere and for much longer periods.

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LEGENDS

Fig. 1. Variation with time of the total potential energy in the volume, bounded by the perimeter of the grid and the 100-mb and 25-mb pressure surfaces.

Fig. 2. Variation with time of the potential plus internal energy in the layer, bounded by the perimeter of the grid and the 100-mb and 50-mb pressure surfaces.

Fig. 3. Variation with time of the potential plus internal energy in the layer, bounded by the perimeter of the grid and the 50-mb and 25-mb pressure surfaces.

Fig. 4. Vertical transport rate of enthalpy through the 100-mb (solid line) 50-mb (dashed line) and 25-mb (dotted line) surfaces. Positive values denote downward transport of enthalpy.

Fig. 5. Rate of conversion of kinetic to potential energy within the 100-50-mb layer (solid line) and within the 50-25-mb layer (dashed line). Negative values represent conversion from kinetic to potential energy.

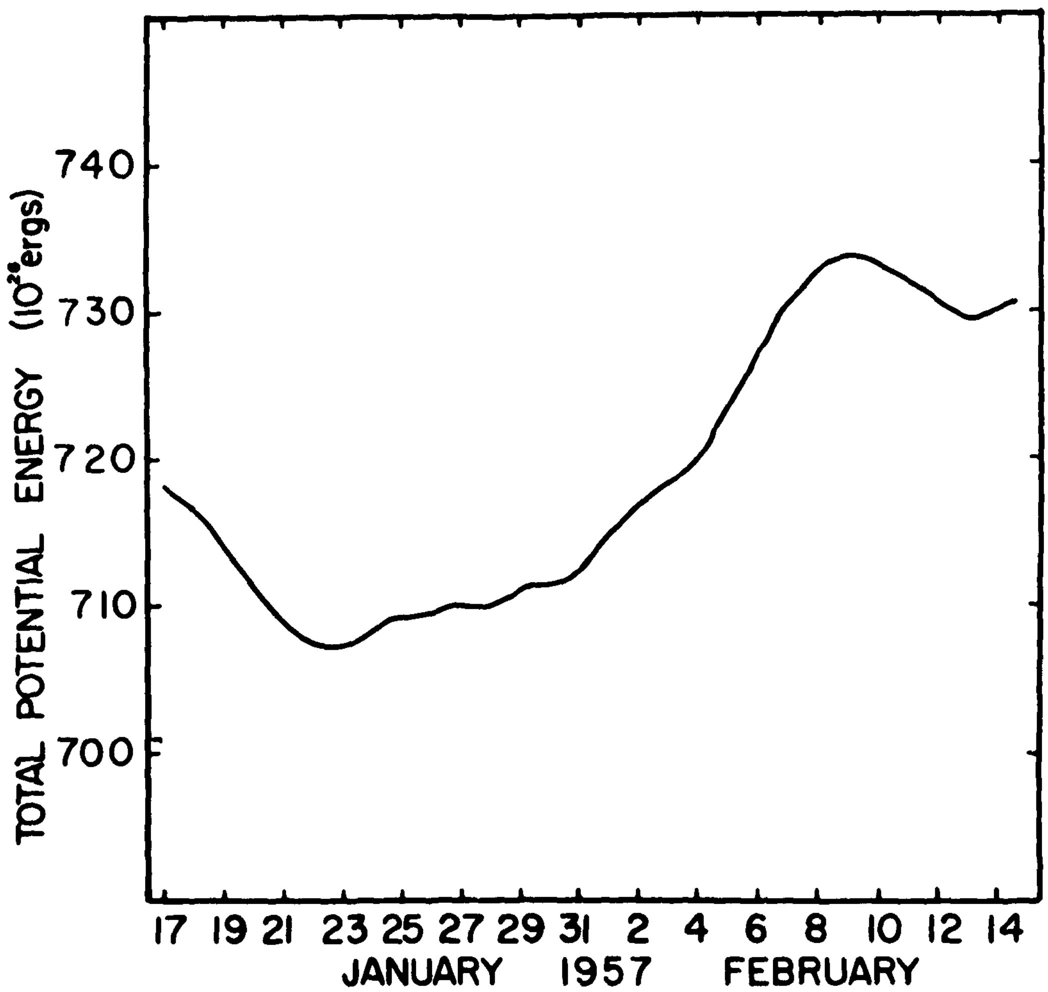

TOTAL POTENTIAL ENERGY (10^{28} ergs)
740
730
720
710
700
17 19 21 23 25 27 29 31 2 4 6 8 10 12 14
JANUARY 1957 FEBRUARY

FIGURE 1

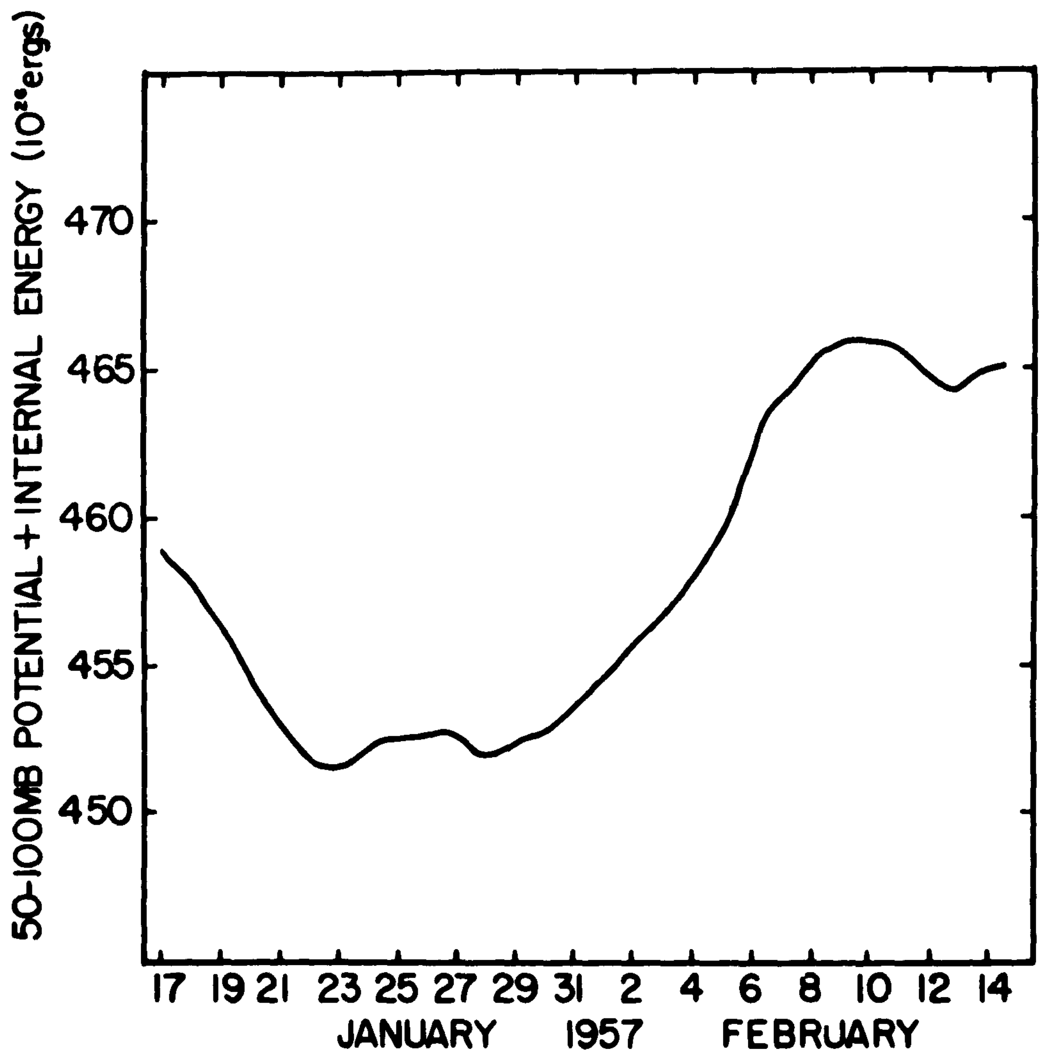

50-100MB POTENTIAL + INTERNAL ENERGY (10^{26}ergs)
470
465
460
455
450
17 19 21 23 25 27 29 31 2 4 6 8 10 12 14
JANUARY 1957 FEBRUARY

FIGURE 2

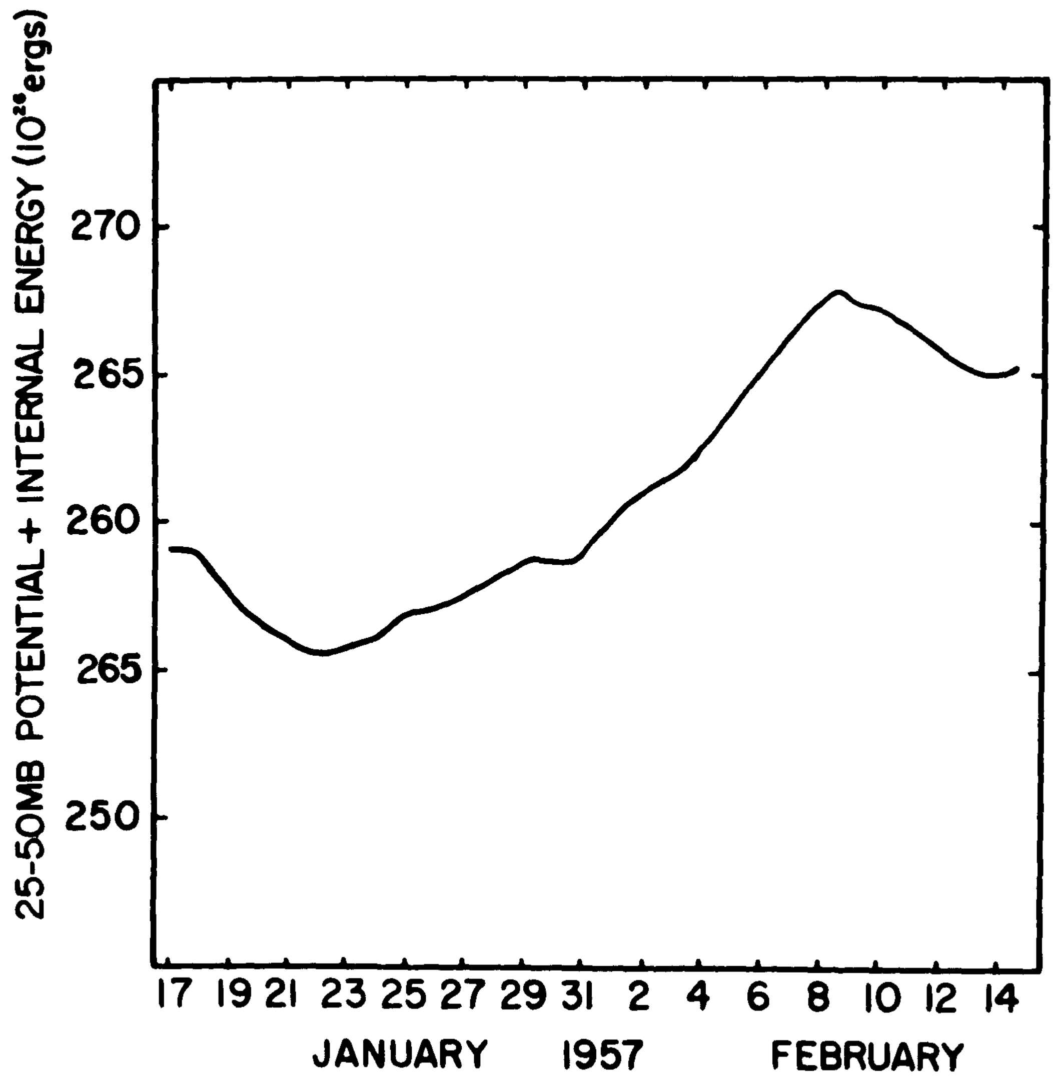

25-50MB POTENTIAL + INTERNAL ENERGY (10^{26}ergs)
270
265
260
265
250
17 19 21 23 25 27 29 31 2 4 6 8 10 12 14
JANUARY 1957 FEBRUARY

FIGURE 3

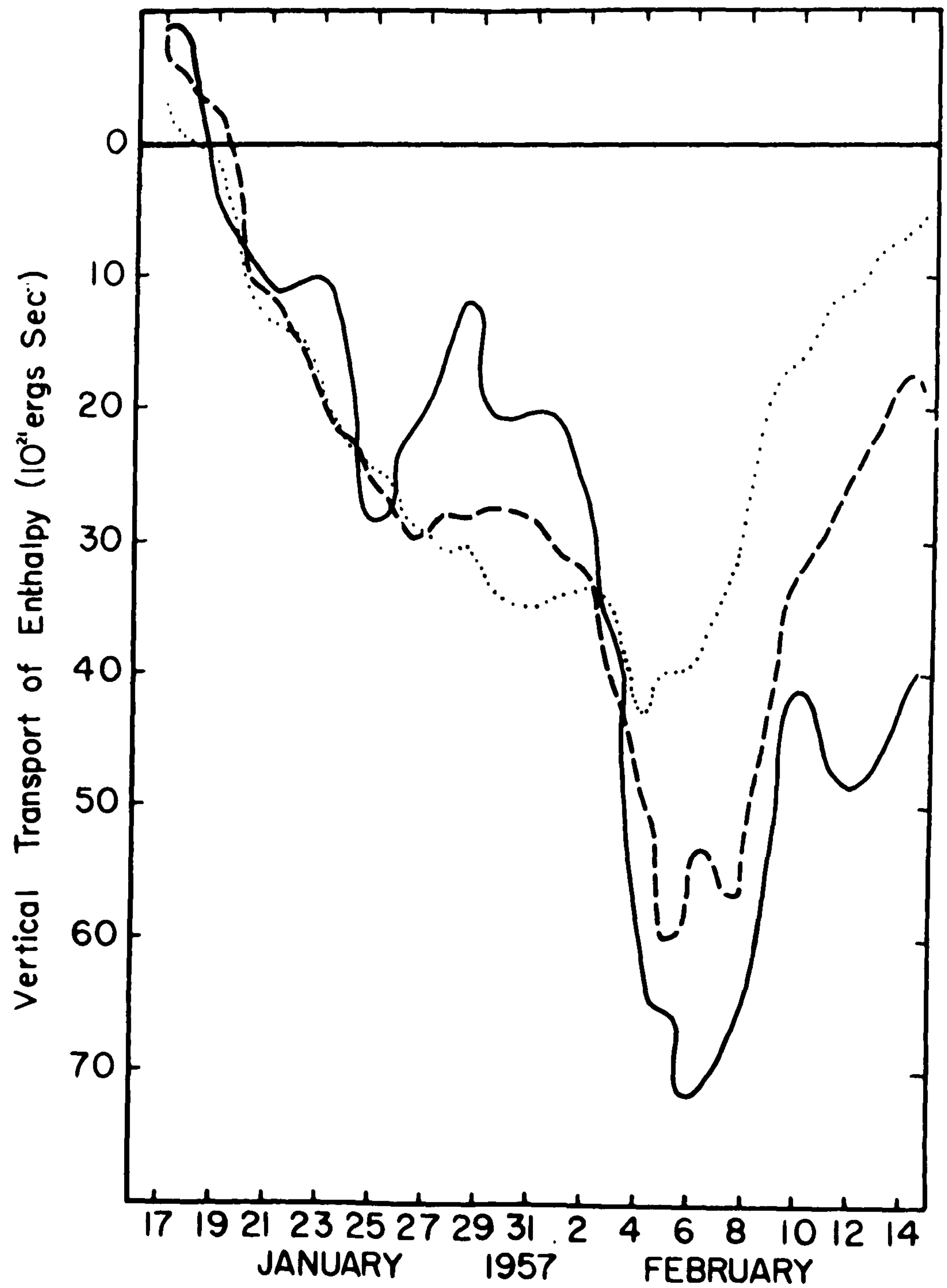
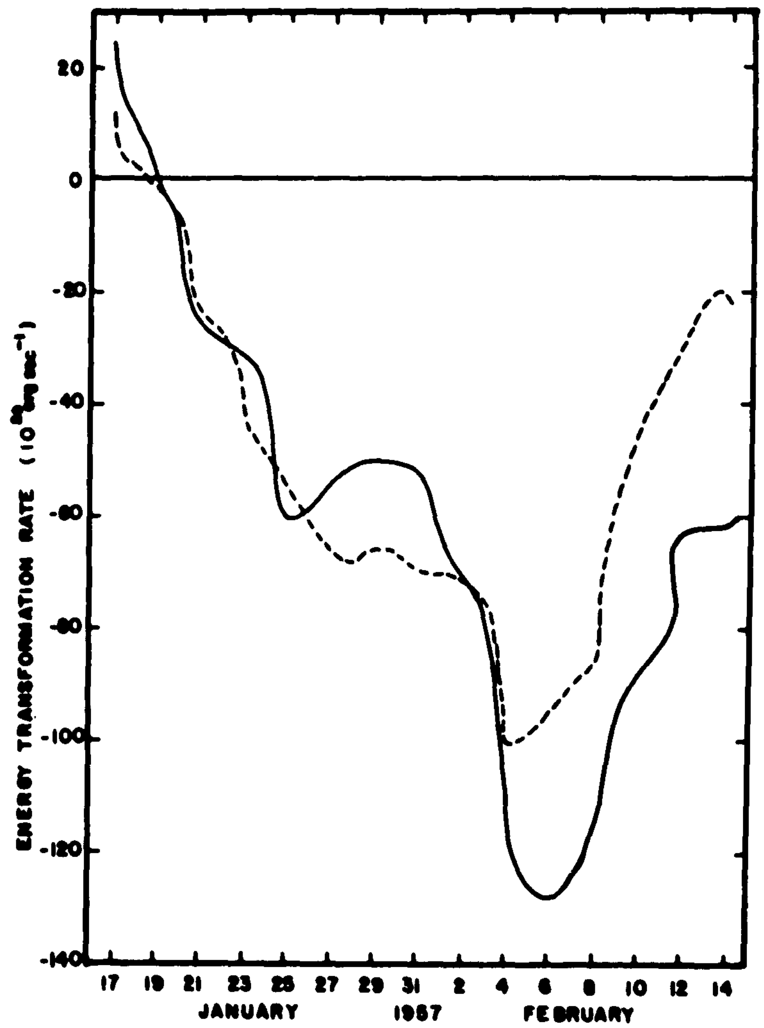

Vertical Transport of Enthalpy (10^{21} ergs Sec^{-1})
0
10
20
30
40
50
60
70
17 19 21 23 25 27 29 31 2 4 6 8 10 12 14
JANUARY
1957
FEBRUARY

FIGURE 4



ENERGY TRANSFORMATION RATE (10^{20} erg sec^{-1})
20
0
-20
-40
-60
-80
-100
-120
-140
17 19 21 23 25 27 29 31 2 4 6 8 10 12 14
JANUARY
1957
FEBRUARY

FIGURE 5

AF Cambridge Research Laboratories, Bedford, Mass.
THE POTENTIAL-ENERGY BUDGET OF THE STRATOSPHERE OVER NORTH AMERICA DURING THE WARMING OF 1957, by M. A. Lateef, June 1963. 23 pp.
AFCRL-63-689 Unclassified report.
This paper discusses the magnitudes of the processes involved in the potential and internal energy balance of the lower stratosphere (100-25 mb) over North America during the warming of mid-January to mid-February, 1957. It is shown that horizontal and vertical flux of energy across the boundary surfaces, contributed significantly to local changes in the potential plus internal energy of the limited atmospheric region under study.

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2. Stratosphere

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